ANALYSIS, DESIGN, AND EVALUATION OF MAN-MACHINE SYSTEMS

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INTRODUCTION

The final goals of prostheses research are to develop artificial limbs controlled naturally by the amputee's motor intents and responding functionally like the natural limbs. However, most prostheses in present use are still far from the goals in spite of recent advanced technologies of robot manipulators. Progress in prostheses requires a more intimate cybernetic interface between amputees and artificial limbs, and a clearer understanding of neuro-muscular-skeletal system controlling natural limbs.

Now skeletal muscle is not simply a generator of contractile force, but also is a noncontractile visco-elastic component. The dynamical properties vary depending on the contractile force, the length of muscle, and its velocity of shortening (Douben, 1980). And the stretch reflex pathway regulates muscle length and provides load compensation in opposition to changes in mechanical load. This is called "follow-up servo hypothesis". Recently an alternative idea has been demonstrated experimentally that motoservo actions are effective in compensating for variations in the mechanical stiffness (impedance) of skeletal muscle (Houk, 1979). Impedance is a term used to describe a variety of different manipulation tasks. Some examples are opening a door, handling eggs, and wiping a pane of grass. Common to all these tasks is that the mechanical impedance (or compliance) of a manipulator defines reaction or contact forces occurring during the action (Mason, 1981). It is suggested from the above that it is of great importance to find out the method effectively to regulate the mechanical impedance in the control of posture and movement (Hogan, 1980).

Hogan (1982) indicated that modulation of joint stiffness was accomplished via coactivation of antagonist muscle groups and discussed a trade-off between antagonist coactivation and metabolically efficient stimulation in postural stabilization through optimization techniques. But, the variability of the stiffness of muscle with activation level was not discussed. The stiffness of muscle primarily defines the static behavior, i.e., equilibrium states of the musculoskeletal system, while rather the compliance of muscle defines the transient behavior of the system. We will emphasize the role of the variable compliance of muscle. This paper shows that the mathematical model derived from the visco-elastic properties of the neuromuscular system has a bilinear form and that from some simulation experiments, the implementation of the bilinear structure as an interface in the human-prosthesis system will lead to considerable improvements in the amputee's control ability.

MATHEMATICAL MODEL

Dynamic Models of Musculoskeletal System

The skeletal muscle is activated by the impulse of motoneurons. The relation between the nerve impulse and the contractile force of muscle is consisted of very ingenious and complicated mechanisms. The macroscopic mechanical properties of muscle can be represented as two fundamental functions of length-tension curves and force-velocity curves (Douben, 1980).

Fig.1 shows the relation between length and tension at various levels of activation under isometric contractions that each muscle length is held constant. Points 100 are rest length and maximum tension respectively. The tension developed by the muscle increases with levels of activation as well as depends on the length of muscle. The level of activation is determined by the impulse rate and recruitment of motoneurons and can be regarded as the input to muscle. Force-Velocity of shortening curves at various levels of activation are shown in Fig.2. Muscle force decreases inversely proportional to the velocity of
contraction as well as increases with the levels of activation.

To obtain a mathematical model, it is assumed that muscle force is in proportion to level of activation (0 ≤ a ≤ 1; normalized by the maximum). Then, the force \( F \) can be given by

\[
F = a \cdot g(L, V)
\]

(1)

where \( g(L,V) \) is a nonlinear function which represents the curves (solid lines) under the maximum level of activation in Fig.1 and Fig.2. Approximating \( g(L,V) \) by Taylor expansion around rest length \( L_{0} \) and the velocity of contraction \( V = 0 \), and neglecting the second terms and upwards, the linear relation is obtained by

\[
g(L,V) = g(L_{0},0) + \frac{\partial g}{\partial L} (L - L_{0}) + \frac{\partial g}{\partial V} V \bigg|_{V=0}
\]

(2)

where \( f_{0} \) is the maximum tension at isotonic contraction \( (V=0) \), \( \dot{x} \) is the relative length of muscle \( (x=0 \) at rest length and \( x > 0 \) is shortening), \( k_{1} \) is velocity of shortening, and \( k_{2} \) and \( b_{1} \) are positive constants. Substituting (2) into (1) yields

\[
F = u_{c} + b_{1} \cdot x - b_{2} \cdot \dot{x}
\]

(3)

where \( u_{c} = a \cdot f_{0}, k_{1} = k_{1} / f_{0}, b_{1} = b_{1} / f_{0} \). This is called viscoelastic model of muscle, but note that viscous and elastic coefficients are not constant and in proportion to the contractile force \( u \).

A viscoelastic model about an elbow joint is shown in Fig.3, where the forearm and hand will be regarded as a rigid link rotating about a fixed axis. By assuming that the flexor and extensor have same properties and the moment arm \( d \) is angle-independent, the muscle torques \( T_{f} \) and \( T_{e} \) about the joint are given by

\[
T_{f} = d(u_{c} + ku_{f} \dot{\theta} - bu_{f} \ddot{\theta})
\]

(4)

\[
T_{e} = d(u_{c} + ku_{e} \dot{\theta} + bu_{e} \ddot{\theta})
\]

(5)

where \( k = k_{1}d, b = b_{1}d \) and the joint angle is defined as \( \theta \) in the vertically upright position and is positive toward flexion (see Fig.3). \( u_{f} \) and \( u_{e} \) are the contractile forces of the flexor and extensor, and are bounded by

\[
0 \leq u_{f} \leq u_{fmax}, \quad 0 \leq u_{e} \leq u_{e max}
\]

(6)

where \( u_{fmax} \) and \( u_{e max} \) are maximum contractile forces. Then the dynamic equation in terms of the horizontal rotational of the forearm is obtained as follows.

\[
I \ddot{\theta} = u_{f} - u_{e} - (u_{f} + u_{e}) k \theta - (u_{f} + u_{e}) b \ddot{\theta}
\]

(7)

where \( I \) is the moment of inertia.

Now the system in the following form is termed bilinear,

\[
dX/dt = AX + \sum_{k=1}^{n} B_{k} u_{k} X + C u
\]

(8)

where \( X \) is an n-dimensional state vector, \( u \) is an \( n \)-dimensional control vector, \( A \) is an \( n \times n \) constant matrix, \( B_{k}(k=1,...,n) \) is an \( n \times n \) constant matrix, and \( C \) is an \( n \times n \) constant matrix. It is said that the bilinear system can offer better performance than the linear system (Mohler, 1973). (7) can be rewritten in the bilinear form. It should be noted that the sum \( u_{f} + u_{e} \) of the contractile forces controls
system parameters while the difference \( u_c - u_e \) controls the driving torque about a joint.

The viscoelastic model which are proposed hitherto are as follows.

\[
\frac{d\theta}{dt} = u_c - u_e - (u_r + u_e)k \theta - b \theta
\]  
(9)

\[
\frac{d\theta}{dt} = u_c - u_e - k \theta - b \theta
\]  
(10)

Eq.(9) (Hogan, 1984) is based on the assumption that only the stiffness of muscle depends on the contractile forces, while eq.(10) (Saw, 1976) is based on the assumption that the stiffness and viscosity of muscle are constant independent of the contractile forces.

**System Structure**

**Equilibrium point.** Substituting \( \dot{\theta} = 0 = \ddot{\theta} \) in (7) and (9), the equilibrium joint angle is given by

\[
\theta_e = \frac{u_c - u_e}{k(u_r + u_e)} = \frac{1 - \delta}{[k(1 + \delta)]} 
\]  
(11)

\( \delta = u_e/u_r \), \( u_r = 0 \)

Note that the equilibrium point is determined by the ratio between the contractile force of flexor and the one of extensor (Hogan, 1980). On the other hand, the equilibrium point of (10) is given by

\[
\theta_e = (u_c - u_e)/k
\]  
(12)

and is determined by the difference between the contractile forces.

**Variable structure.** The eigenvalue of the linear system (10) is determined completely by the parameters \( k \) and \( b \) while that of the bilinear systems (7) and (9) depends on the sum \((u_r + u_e)\) of the contractile forces. It is seen that from the constraints (6) and \( b > 0 \), \( k > 0 \), the eigenvalues of (7) and (9) are stable real roots or complex roots. Therefore, the state trajectories are able to have stable focus or nodes according to the following criteria:

**Model (7):**
1. \( 0 \leq u_c - u_e \leq 4k \theta/d\theta \) : stable focus
2. \( 4k \theta/d\theta \leq u_c - u_e \leq \max \theta \text{max} \) : stable node

**Model (9):**
1. \( 0 \leq u_c - u_e \leq \delta d\theta/4k \) : stable node
2. \( \delta d\theta/4k \leq u_c - u_e \leq \max \theta \text{max} \) : stable focus

Thus the bilinear model is characterized by the variable structure that the node of system can be changed by the contractile forces \( u_c - u_e \).

The simulation results of three models are shown in Fig.4 to make clear the difference. The upper correspond to the contractile levels of the flexor \( u_r \) and the extensor \( u_e \), and the lower are the state trajectories where \( \theta \) denotes the deviation from each equilibrium point. Fig.(a) shows that the response of the linear model (10) is a bang-bang control form. The switching time must be exactly set up to pass through the equilibrium point. Fig.(b) and (c) simulate the bilinear models and show three levels of \( u_r \) and \( u_e \) under the condition that \( u_c - u_e \) remains same. All the trajectories converge to the same equilibrium point (the origin). It should be noted, however, that the system node varies. In our model (7), the state trajectory changes from stable focus to stable node as \( u_c - u_e \) increases, which means that the system is able to turn more damping.

Inversely, the state trajectory of the model (9) changes from stable node to stable focus as \( u_c - u_e \) increases, which means that the system is able to turn more oscillatory.

The state trajectories of the bilinear model

![Fig.4 Simulation Results of Three Types of Models (I=0.003, d=1.0, k=0.2, b=0.05)](417)
(7) are illustrated in Fig.5 where the switching time is shifted in each iteration keeping same \( u_f / u_e \) and \( u_f - u_e \). It is seen that all the trajectories proceed to the same equilibrium point determined by the ratio of the contractile forces without regard to the shifts of the switching time. Therefore, when we rotate the forearms to the desired position, we need not exactly to set up the time to switch the contractile forces of the flexor and extensor unless the transient response is important.

As an example of actual motion, Fig.6 shows EMG pattern from flexor (biceps) and extensor (triceps) during a ballistic horizontal movement of the forearms. The flexor is activated first and then the flexor and extensor are coactivated. This shows that the human subject positively utilizes the variable structure of the neuromuscular system.

In this paper, we will concentrate our discussion on the design problem of the control system about the elbow joint. It is general that myoelectric prostheses available today adopt on-off control or proportional control. The former uses the myoelectric signal only to turn on or off the actuator of the prosthesis. The latter uses the processed myoelectric signal directly as command signals of the actuator, and then the lock-unlock control is accomplished by high and low levels of cocontraction of the antagonist muscles or by a third signal acquired from a motion switch (Jacobson, 1982).

Here we stress again the role of mechanical impedance about a joint. The amputee should be able to regulate the control properties of the prosthesis through impedance modulation (see Fig.8(a)). Therefore, the bilateral structure is added as a compensatory loop to the proportional control system (see Fig.8(b)).
Control Properties of Human-Prosthesis System

![Diagram of Control Properties](image)

The mathematical model which simulates horizontal movements of a prosthetic arm is given as follows.

\[ I \ddot{\theta} + B_0 \dot{\theta} = T \]

where \( I \) is moment of inertia, \( B_0 \) is coefficient of viscosity about the joint, and \( T \) is joint torque. A couple of controllers were tested. One is bilinear as shown in Fig.8(b) and another is linear within a broken line. In the bilinear controller, the driving force and system parameters can be controlled independently via the difference and sum of the flexor and extensor.

**Tracking Tests**

A computer-controlled tracking test was performed. The human subject sat before the graphic display and was instructed to move the forearm from current position to desired position as fast as possible. The desired positions were shifted by random angle (within 80°) and time span (4-8 sec.). Tracking time was 50 sec. and three normal subjects were used. A performance index should include all three scores of error, response time and control cost. The following performance scores were computed: (1) Integrated Squared Error (ISE), (2) Integrated Absolute Error (IAE), (3) Integrated Time Absolute Error (ITAE), (4) Integrated Control Cost (ICC),

\[ ICC = \int (u_T + u_e)^2 dt \]

The averaged scores and standard deviations of ten trials after training are shown in Fig.10. Three scores (ISE, IAE, ITAE) of the bilinear controller, except ICC, were lower by 30% - 50% than the linear controller. This suggests that the bilinear controller can develop the amputee's tracking abilities. On the other hand, however, the control cost (ICC) increased largely. Although the subject's EMG levels still remain not exceeding about 40% of the maximal voluntary contraction, we have to solve the trade-off between energy consumption and controllability.

**Position Control Experiment**

It is to be desired that the amputee can certainly make a prosthesis reflect his motor intents and also perform a given task with the least amount of concentration and effort. A variety of different types of tasks could be used to evaluate control performance. Here we use tracking tasks which are often used in the field of manual control.

A block diagram in Fig.9 illustrates the experimental arrangement of man-machine system controlled by myoelectric signals with visual feedback. The EMG signals from biceps and triceps were used to provide a command signal. The subject's forearm was fixed on the horizontal table keeping angle of the elbow 90°. The subject generates the EMG signals through isometric contraction. A pair of differential electrodes on each muscle were 15 mm diameter disks and were separated by 20 mm. After full wave rectification the EMG signals were processed by a couple of low-pass analog filters (f_c = 4 Hz, first-order), the outputs of which are command signals \( u_T \) and \( u_e \).

The forearm was drawn on the graphic screen. The solid line is a current position, and the broken line is an desired position. The computation time for the controller and the mathematical model of prosthesis was negligible small. The forearm on the display was rotated corresponding to the output \( u_T \) from the computer. Therefore, the human subject can rotate the forearm by his myoelectric signals.

Fig.9 A Block Diagram of Experimental Arrangement

![Diagram of Experimental Arrangement](image)
amplitude, while the bilinear system yields sharp switching and large amplitude, i.e., about three times as large as the linear system. This leads to the difference in speed of response between both systems.

Linear system never allows the control to change the system parameters. Therefore, when the large control is inputted to get a short rising time, it becomes more difficult to predict the switching point, which will lead to the overshoot. In contrast to this, the multiplicative control of bilinear system can be used to change the system to a large damping system at any time, which may result in the large input at the first half.

CONCLUSION

It is the important property of bilinear system that the control input is able to vary the system structure (node), which is a very important feature. Modification of viscoelastic property about the joint by coactivation of antagonist muscles increases the flexibility of the system and plays a role to facilitate the open loop control of movement (Houk, 1974; Hogan, 1984). In this paper, it was shown that the position control could be improved largely by adding the bilinear structure to the interfaces in human-prosthesis system.

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